Assignment 16

R 13.1 Professor Amongus has shown that a decision problem L is polynomial-time reducible to an NP-complete problem M. Moreover, after 80 pages of dense mathematics, he has also just proven that L can be solved in polynomial time. Has he just proven that P=NP? Why or why not?

Answer:

No, he has not proved p=NP. So, to prove P=NP, he has to show one of the followings

1. L is an NPC problem or M is reducible to L
2. L is an NPH problem.

If he is able to prove any of the above and all NP problems can be reduced to L which can be solved in polynomial time then he will be able to prove P=NP.

R 13.3 Show that the problem SAT is NP-complete; SAT takes an arbitrary Boolean formula S as input and asks if S is satisfiable,.

Answer:

Algorithm SAT2SubSetSum(S)

R<- new Empty List

R.Insert(5)

if CheckBoolGate(S)= true then

return (R, 5, 5)

else

return (R, 2, 2)

We have reduced SAT to SubSetSum problems and it runs in polynomial time. SubSetSum runs in polynomial time and it is in NPH and NP. So, SAT ∈ NPC

R-13.13 Is there a subset of the numbers in {23, 59, 17, 47, 14, 40, 22, 8} that sums to 100? What about 130? Show your work.

Answer:

Algorithm CheckSubSetSum(NumberSet, Target)

return (NumberSet, Target, Target)

(47+23+22+8=100)

(59+40+14+17 =130}

A. Prove that the Set-Partition decision problem is a member of class NP. Set-Partition is defined as follows:

Set-Partition: Given a set S of integers, does there exist a partitioning of S into two disjoint partitions, such that the sum of the elements of both partitions is the same?

Hint: To be a partitioning, each element of S must be in either P1 or P2, but not both. Two partitions, P1 and P2 are disjoint if and only if no element S is a member of both P1 and P2. For example, suppose that S1 = {3, 6, 3}, then S1 can be partitioned into two partitions P1={3,3} and P2={6} whose sums are equal (6). However, S2={3, 5} cannot be partitioned in a way where the sums of two partitions are equal. Thus S1 is a member of the Set-Partition language, but S2 is not.